Planetary Migration and Extrasolar Planets in the 2/1 Mean-Motion Resonance

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ABSTRACT

We analyze the possible relationship between the current orbital elements fits of known exoplanets in the 2/1 mean-motion resonance and the expected orbital configuration due to migration. We find that, as long as the orbital decay was sufficiently slow to be approximated by an adiabatic process, all captured planets should be in apsidal corotations. In other words, they should show a simultaneous libration of both the resonant angle and the difference in longitudes of pericenter.

We present a complete set of corotational solutions for the 2/1 commensurability, including previously known solutions and new results. Comparisons with observed exoplanets show that current orbital fits of three known planetary systems in this resonance are either consistent with apsidal corotations (*GJ876* and *HD82943*) or correspond to bodies with uncertain orbits (*HD160691*).

Finally, we discuss the applicability of these results as a test for the planetary migration hypothesis itself. If all future systems in this commensurability are found to be consistent with corotational solutions, then resonance capture of these bodies through planetary migration is a working hypothesis. Conversely, If any planetary pair is found in a different configuration, then either migration did not occur for those bodies, or it took a different form than currently believed.

Key words: celestial mechanics, planets and satellites: general.

1 INTRODUCTION

It is well known that extrasolar planets are not where we imagined. Classical planetary formation theories based on planetesimal accretion and core-instability for the giant planets predict bodies in quasi-circular orbits and semimajor axes a far from the star. For solar type stars, the minimum semimajor axis is about 4 AU, which is the distance where non-rocky volatile elements can condense and accrete. However, many exoplanets do not follow this rule, but are found in highly eccentric orbits with $a < 1$ AU.

Two options have been proposed to explain this dilemma. In the first, it is assumed that present cosmogonic theories are in fault, or at least they followed different routes in practically all other planetary systems (thus making our own Solar System a very particular case). In the second, exoplanets really did form far from the central star, but suffered a posterior decay in their semimajor axes towards their present sites. Thus was born what is usually referred to as the "Hypothesis of Planetary Migration". However, in order for migration to be a real theory and not just a simplistic escapade, two conditions must be met: *(i)* the existence of a plausible driving mechanism to explain the alleged decay in orbital energy, and *(ii)* concrete evidence that exoplanets did undergo such an evolution.

Two different driving mechanism have been presented in the last few years. The first (Murray et al. 1998) is based on the interaction of the planets with a reminiscent planetesimal disk, and works in the same manner as migration of the giant planets in our Solar System (Fernandez and Ip 1984, Hahn and Malhotra 1999). However, this mechanism does not seem to be sufficiently efficient. First, it requires a very large disk mass, of the order of 0.1 $M_\odot$, to explain an orbital decay of several astronomical units. Second, it is not completely evident that multi-planet systems should in fact undergo a simultaneous decrease in semimajor axis. Recall that in our system, Jupiter is believed to have suffered decrease in orbital energy, while Saturn, Uranus and Neptune have increased their value of a.

The second proposed mechanism is the interaction of the planets with the gaseous disk. The existence of disk torques cause a transfer of energy and angular momentum from the planet to the gas and, if the disk parameters are chosen correctly, then the solid body should undergo a negative migration (i.e. $\dot{a} < 0$). Several simulations have been performed in recent years (e.g. Snellgrove et al. 2001, Nelson and Papaloizou 2002, Kley 2003, Papaloizou 2003) and these seem to indicate that the mechanism works reasonably well. However, certain aspects of this process are also problematical. For some disk parameters the migration can be positive, leading to an increase in semimajor axis, which is just the opposite desired result. In some other cases, the orbital energy may even exhibit

random-walks with no secular variation. Nevertheless, it seems that this mechanism is the most probable process to explain migration.

Having found a plausible process for the orbital decay, we must now search for evidence that this really occurred in the ex-oplanets. This question is particularly important, since such large-scale *inward* migration did not happen in our case. A possible solution is to find a particular orbital characteristic of the extra-solar bodies, intimately related to migration, which can be used as (at least) indirect evidence of this process. Our Solar System presents two cases of confirmed migration (be it outward or in-ward): the giant planets and planetary satellites. As mentioned be-fore, the outer planets migrated due to interaction with a remnant planetesimal disk, while many of the regular satellites of these same planets evolved due to tidal effects of the central mass. In this lat-ter case, we know that an important consequence of the migration was capture of the satellites in exact mean-motion resonance (e.g. Colombo et al. 1974, Yoder 1979). A well known example is given by the Galilean satellites of Jupiter. It is known that these configu-rations cannot be explained solely with gravitational perturbations, but only through resonance trapping under the effects of an exte-rior non-conservative force. The case of our outer planets is differ-ent. It seems that they are not exactly in resonance due to certain random-walk characteristics of the driving mechanism itself (Hahn and Malhotra 1999). Thus, we can conclude that although migra-tion does not always lead to resonance trapping, the existence of massive bodies in exact mean-motion resonance can only be ex-plained via a migration mechanism.

The fact that our planets did not suffer a significant smooth inward migration is consistent with the fact that none of them are trapped in resonance. What about the exoplanets? A good evidence in favor of migration would then be to analyze whether plane-tary systems do show mean-motion resonant relations. Of the 13 presently known planetary systems, including both confirmed and un-confirmed cases, we restrict ourselves to those systems where the ratio in semimajor axes is sufficiently small to assure signif-icant gravitational interaction between the bodies. Choosing this limit to be $a_2/a_1 = 3$, we find that 6 systems satisfy this condi-tion. These are: *GJ876*, *55Cnc*, *47Uma*, *HD82943*, *HD160691* and *Ups And*. Of these, five are believed to be in the near vicinity of mean-motion resonances, while *Ups And* is in a secular resonance.

Although this proportion is very significant, it must be consid-ered with care. Recent data analysis (Mayor et al., private com-munication) shows that the published orbit of *47 Uma* may be questionable, and there is no general agreement if the system is near the 7/3 or 5/2 mean-motion resonances. Doubts also exist for *HD160691*, and the second planet is not yet confirmed. In view of this debate, and considering the intrinsic errors in orbital fits, the mere proximity of these systems to mean-motion resonances is not evidence enough for migration, especially considering that there may be a natural tendency of researchers to place planets in commensurabilities even though they may not be near enough. For these reasons, we feel a more detail analysis is necessary.

This manuscript undertakes such a analysis. In Section 2 we present new results on the location and characteristics of corota-tions in the 2/1 resonance. Section 3 discusses the problem of plan-etary migration from the point of view of Adiabatic Invariant theory and we show that only corotational type configurations can appar-ently be expected in trapped planets. A comparison between these solutions and the current orbits of three exoplanetary systems are detailed in Section 4. Finally, conclusions close the paper in Section 5.

2 GENERAL APSIDAL COROTATIONS FOR THE 2/1 MEAN-MOTION RESONANCE

Suppose two planets of masses m_1 and m_2 in coplanar orbits around a star of mass $M_0 \gg m_1, m_2$. Let a_i denote the semi-major axis of the ith-planet ($i = 1, 2$), e_i the eccentricity, λ_i the mean longitude and ϖ_i the longitude of the pericenter. All orbital elements correspond to Poincaré canonical relative coordi-nates (see Ferraz-Mello et al. 2004), which differ from the classical star-centered orbital elements in the second order of the planetary masses. We will suppose $a_1 < a_2$, thus the subscript 2 will corre-spond to the outer orbiting body.

We now assume that both secondary masses are located in the vicinity of a resonance such that their mean motions n_i satisfy the relation $n_1/n_2 \simeq (p + q)/p$. Both p and q are small integers and q is usually referred to as the order of the resonance. The name "apsidal corotation" (see Ferraz-Mello et al. 1993) is used to denote the simultaneous libration of both resonant angles:

$$
\begin{aligned}
\theta_1 &= (p+q)\lambda_2 - p\lambda_1 - q\varpi_1 \\
\theta_2 &= (p+q)\lambda_2 - p\lambda_1 - q\varpi_2.
\end{aligned}
\tag{1}
$$

It is straightforward to write $\theta_2 - \theta_1 = q(\varpi_1 - \varpi_2) = q\Delta\varpi$, thus an apsidal corotation can also be identified with the libration of both θ_1 and the difference in longitudes of pericenter.

Once the short-period perturbations (associated with the syn-odic period) are eliminated by an averaging process, the resulting system is a two degree of freedom problem and can thus be speci-fied by two angular variables, for example, $(\theta_1, \Delta\varpi)$. Their canon-ical conjugates are given by:

$$
\begin{aligned}
I_2 &= \frac{1}{q}L_2\left(1 - \sqrt{1 - e_2^2}\right) \\
I_1 &= I_2 + \frac{1}{q}L_1\left(1 - \sqrt{1 - e_1^2}\right).
\end{aligned}
\tag{2}
$$

The quantity $L_i = m_i'\sqrt{\mu_i a_i}$ is the modified Delaunay momenta related to the semimajor axis in Poincaré variables (see Laskar 1991, Ferraz-Mello et al. 2004), $\mu_i = G(M_0 + m_i)$, and G is the gravitational constant. The factor m_i' is a reduced mass of each body, given by:

$$
m_i' = \frac{m_i M_0}{m_i + M_0}.
\tag{3}
$$

It is easy to see (e.g. Michtchenko and Ferraz-Mello 2001) that the total planar angular momentum of the system, in itself an integral of motion, is given by:

$$
J_{\text{tot}} = L_1 + L_2 - I_1.
\tag{4}
$$

Similarly, the complete averaged Hamiltonian of the system can be expressed in terms of the orbital elements as:

$$
F = -\sum_{i=1}^{2} \frac{\mu_i^2 m_i'^3}{2L_i^2} - F_1(m_1, m_2, a_1, a_2, e_1, e_2, \theta_1, \Delta\varpi),
\tag{5}
$$

where the disturbing function F_1 denotes the gravitational interac-tion between both planets. Further details can be found in Beaugé and Michtchenko (2003). With this in mind, apsidal corotations can now be constructed from the conditions:

$$
\begin{aligned}
\frac{dI_1}{dt} &= 0 \quad ; \quad \frac{dI_2}{dt} = 0 \\
\frac{d\theta_1}{dt} &= 0 \quad ; \quad \frac{d\Delta\varpi}{dt} = 0.
\end{aligned}
\tag{6}
$$

In other words, they are fixed points of the averaged Hamiltonian

(5). Once the short-period variations are re-introduced, corotations actually correspond to periodic orbits (Hadjidemetriou 2002, Hadjidemetriou and Psychoyos 2003).

It is important to emphasize two points. First, apsidal corotations are zero-amplitude solutions in the averaged problem. In a real system, the planets may in fact undergo finite-amplitude oscillations around these points, thus describing quasi-periodic orbits in real space. The simulations by Lee and Peale (2002) of the *GJ876* show such a behavior. A second, and very important note, is that it is possible that the amplitude of oscillation in $\Delta\varpi$ be sufficiently large to show an actual circulation (and not a libration) of this angle. In other words, the difference in longitudes of pericenter may vary from zero to 360 degrees, although topologically the solution is still an apsidal corotation. Thus, a finite-amplitude corotation is really defined in terms of a separatrix crossing to a different mode of oscillation not present in the unperturbed dynamical system, and not solely in terms of the temporal variation of $\Delta\varpi$.

In a recent study, Beaugé et al. (2003) (hereafter referred to as BFM2003) and Ferraz-Mello et al. (2003) performed a systematic search for different types of corotational solutions in the 2/1 and 3/1 resonances. Among our first results we found that, up to second order of the masses, apsidal corotations only depend on the real masses of the planets through the ratio m_2/m_1. In other words, these solutions are not function of the individual values, and are thus independent of the inclination of the orbital plane of the planets with respect to the observer (as long as both planets share the same plane). This is a very interesting point, since it allows us to bypass the limitations in Doppler orbital fits. Secondly, we also found that these periodic orbits only depend on the semimajor axes through a_1/a_2. Since this ratio is only an indication of the proximity to exact resonance, it is independent of the individual values of the semimajor axes themselves. As a consequence of these properties, we were able to obtain the apsidal corotations as level curves of θ_1, $\Delta\varpi$ and m_2/m_1 in the plane of eccentricities (e_1, e_2), and these results were seen to be extremely general. They are valid for any planetary system, independently of the values of the real masses and the distance from the central star.

For the 2/1 resonance, our results showed the existence of three types of corotational solutions. Aligned apsidal corotations are characterized by equilibrium values of the angles equal to $(\theta_1, \Delta\varpi) = (0, 0)$. Anti-aligned solutions are given by $(\theta_1, \Delta\varpi) = (0, \pi)$. Both these families were previously known by other authors (e.g. Lee and Peale 2002, Hadjidemetriou 2002). However, we also discovered a new type of orbits, called asymmetric apsidal corotations, which were characterized by values of $(\theta_1, \Delta\varpi)$ different from zero or π (see Greenberg 1987 for similar results for the Galilean satellites). Finally, to each value of (e_1, e_2) there seemed to correspond only one equilibrium value of the mass ratio m_2/m_1. Similar results were also noted for the 3/1 resonance.

Due to the inherent limitations of our model, we were only able to detect apsidal corotations with eccentricities up to $e_i = 0.5$. Moreover, we only analyzed cases where $m_2/m_1 > 0.1$. This limit was chosen simply for computational reasons. Recently, however, two new results came to our attention. Numerical studies by Hadjidemetriou and Psychoyos (2003) noted that some eccentricities allowed for two different equilibrium values of m_2/m_1, one of them smaller than 0.1. Secondly, the same authors also found a new type of corotational orbit, characterized by $(\theta_1, \Delta\varpi) = (\pi, \pi)$ for very high values of e_1, e_2.

However, both these studies only analyze a restricted number of initial conditions and do not present general results in the (e_1, e_2) plane. Thus, a more general analysis may be useful. Since
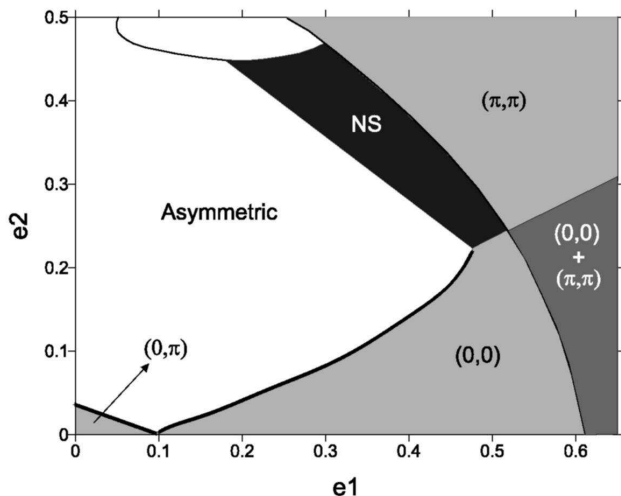

Figure 1. Domains of different types of corotational solutions in the 2/1 mean-motion resonance, as seen in the plane of orbital eccentricities of both planets. NS is the region with no stable solutions. See text for further explanations.

our previous model is not sufficiently adequate for such high eccentricities, for the present paper we adopted a new semi-analytical approach based on the so-called Extended Schubart Averaging (Moons 1994), where the Hamiltonian (5) and conditions (6) are solved numerically. We also lifted the restriction in the minimum mass ratio, thus allowing us to study the general corotational solutions with no restriction whatsoever in their parameters.

Figure 1 shows the plane of eccentricities for the 2/1 resonance, where we have drawn the limits of the domains of all types of solutions. Each is marked by the equilibrium values of the angles, except the asymmetric region, plus a new domain denoted by NS (i.e. No Solution). In this region there are no stable apsidal corotations for any values of the mass ratio. This is due to close encounters, where the potentially equilibrium values of the angles correspond to quasi-collisions between the planets. We will return to this point further on. Note that the (π, π)-corotations are located beyond the collision curve, at very high eccentricities; nevertheless this domain intersects the aligned apsidal corotation regime for relatively low values of e_2. In this intersection, two distinct types of stable solutions (aligned and (π, π)) coexist, although for different values of the masses. Finally, the asymmetric zone is actually divided into two distinct regions which are not connected via a smooth variation of the planetary masses.

The equilibrium values of the resonant angle θ_1 and of the difference in pericenter $\Delta\varpi$ are shown in Figure 2. All types of solutions are shown with the exception of the (π, π)-corotations, which will be discussed further on. This figure is analogous to the results presented in BFM2003, although extended to higher eccentricities. We note two main differences. First, asymmetric apsidal corotations are limited to $e_1 < 0.5$, after which only aligned orbits are possible. Second, the NS region was not detected in our previous work. This was due to the limitations of the analytical model, where our expansion of the Hamiltonian underestimated the values of the function in the vicinity of the collision curve. Nevertheless, all the solutions with $e_1 + e_2 \leq 0.5$ are equal in both determinations.

The level curves of mass ratios for $(0, 0)$, $(0, \pi)$ and asymmetric solutions are shown in Figure 3. Notice that for practically all
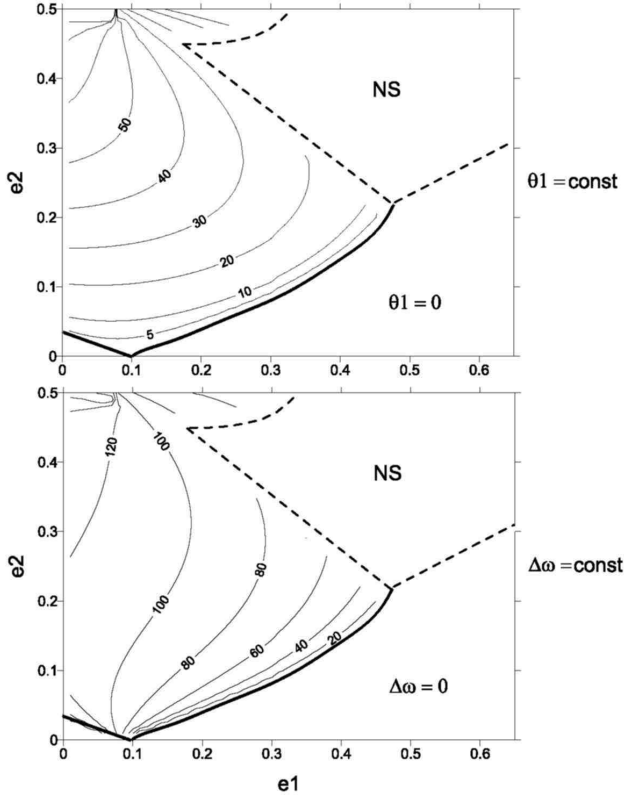
Figure 2. Top: Level curves of equilibrium values of the resonant angle θ_1 for all corotations below the collision curve. Bottom: idem, but for the values of $\Delta\varpi$.

Figure 3. Level curves of constant mass ratios m_2/m_1 for stable corotations in the 2/1 resonance. Note that most values of the eccentricities have two possible solutions. Larger values of the masses are plotted on the top graph, while smaller values are shown on the bottom.

eccentricities there are two distinct equilibrium values of m_2/m_1, in accordance with the predictions of Hadjidemetriou and Psychoyos (2003). Larger ratios are shown on the top graph, while smaller quantities on the bottom plot. Figure 4 shows similar results, now for the (π, π)-corotations.

In order to have a better understanding of the origin of the NS region in Figure 1, we studied the variation in the mass ratios as function of e_2 for a constant value of the eccentricity of the inner planet ($e_1 = 0.3$). This value was chosen so that it intersects the domain of No Solutions, as well as aligned and asymmetric apsidal corotations. Results are shown in Figure 5, where we can see that the boundaries of the NS region are characterized by a coalescence of two values of the mass ratios.

A final data we wish to present at this point is the period of motion of oscillations around apsidal corotations. As early as the analysis of *GJ876* by Lee and Peale (2002), it is known that exoplanets do not necessarily have to be in an exact periodic orbit (i.e. zero-amplitude apsidal corotation), but can exhibit a finite amplitude oscillation around this solution with a certain period. This quantity, at least for the linear approximation, can be determined calculating the Hessian of the Hamiltonian evaluated at each apsidal corotation. However, due to the order of the characteristic equation, this method is very time-consuming and sensitive to numerical errors. For these reasons, in the present work we employed a more numerical approach.

Considering a fixed mass ratio, we first performed numerical simulations of the evolution of the system along the family of periodic orbits, in a manner analogous as presented in Ferraz-Mello et al. (2003). We then calculated a running Fourier analysis of the
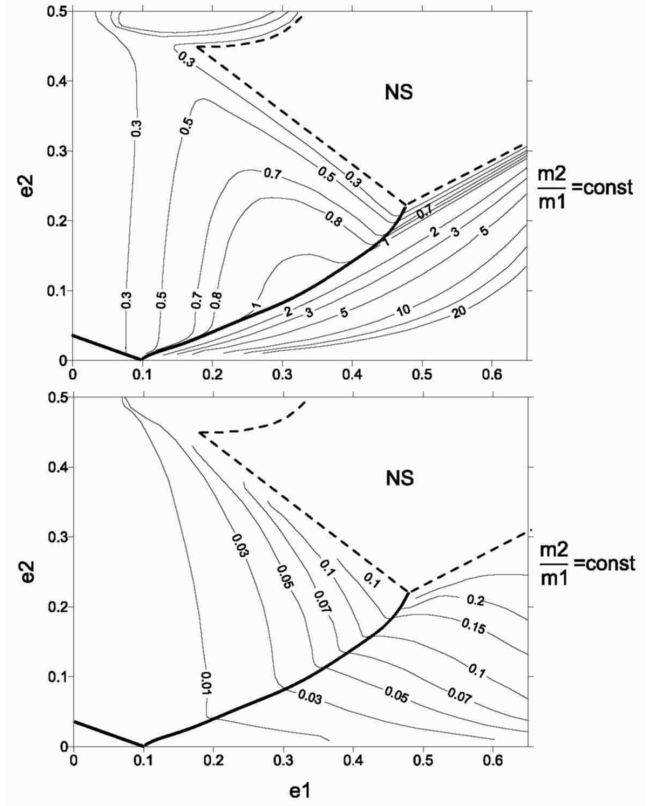
angular variables at given times, and calculated the period τ associated with the largest amplitude. Simultaneously, we also estimated the averaged planetary eccentricities, thus obtaining a relation between τ and e_1. Results are presented in Figure 6, in units of years, for four different mass ratios. These periods correspond to $a_2 = 1$ AU and $m_1 = M_{\mathrm{Jup}}$. It must be noted that the curves have been smoothed, both to eliminate spurious differences between adjacent points, and to soften the separatrix separating symmetric from asymmetric solutions. Thus, individual values must be considered more qualitative than quantitatively correct, although the general trend is fairly accurate.

Even with these notes of caution in mind, the plot still gives valuable information. We can see that the period of oscillation increases for smaller values of the mass ratio, and the rate is practically inverse-linear. Thus, the maximum τ for $m_2/m_1 = 3$ (similar to the *GJ876* system) is about 300 years, while for a mass ratio of 0.5, the maximum period is about six times larger. Second, comparing these results with Figure 3, we note that asymmetric apsidal corotations have much larger periods than symmetric solutions. This characteristic will prove important in later sections.

Finally, as mentioned before, the quantities in the graph correspond to $a_2 = 1$ AU and $m_1 = M_{\mathrm{Jup}}$. For other values of these parameters, the resulting period must be scaled according to:

$$\tau = \tau_0 a_2^{3/2} \left(\frac{m_1}{M_{\mathrm{Jup}}} \right)^{-1} \qquad (7)$$

where τ_0 is the corotational period given in the figure. Thus, although the position of the apsidal corotations are only function of
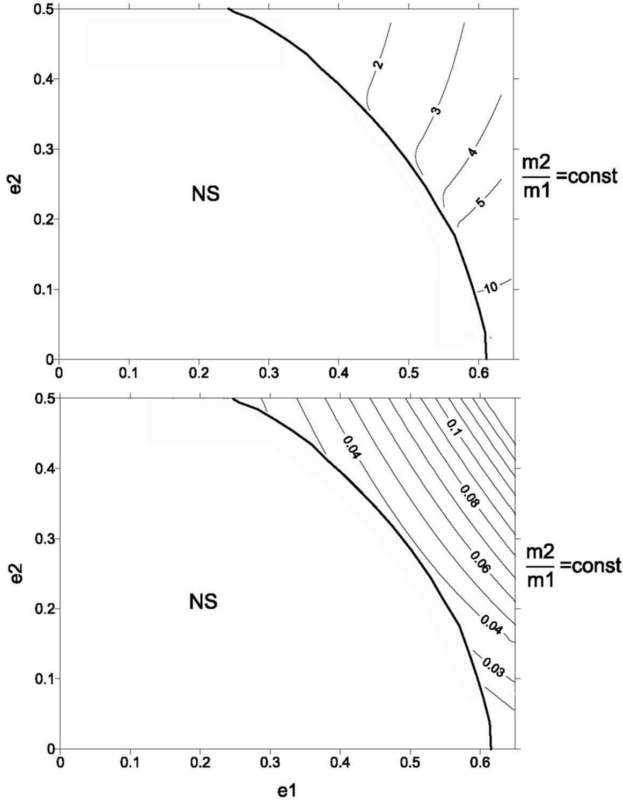
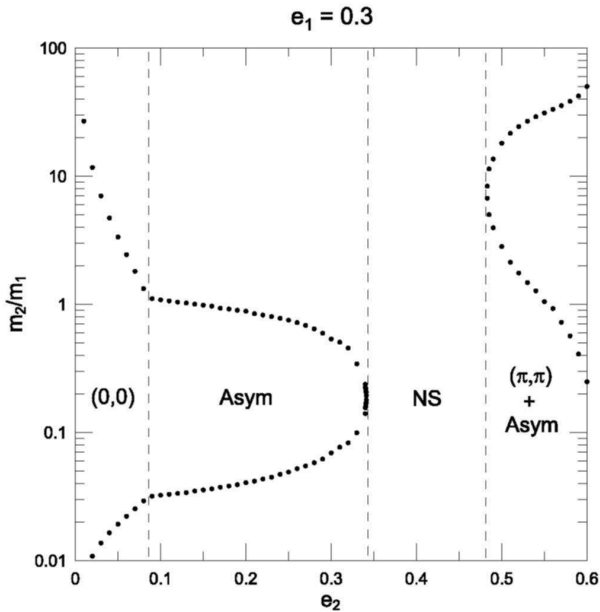**Figure 4.** Idem previous figure, but for the (π, π)-corotations.

Figure 5. Mass ratio of corotations, as function of the eccentricity of the outer body, for two values of e_1. Red shows $e_1 = 0.3$ while black corresponds to $e_1 = 0.5$. Notice that the region of No Solutions appears when both mass ratios per eccentricity fold into a single solution.
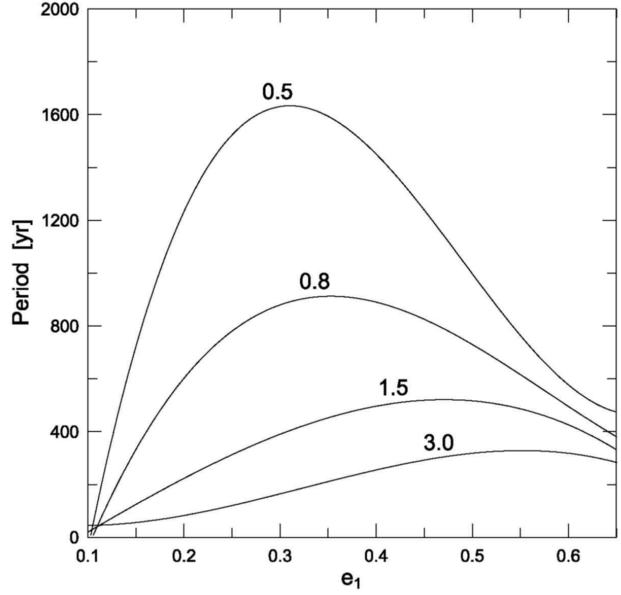

Figure 6. Period of infinitesimal oscillations around corotations, for four different mass ratios, as function of the eccentricity of the inner planet.

m_2/m_1, the periods of oscillation are linearly dependent on the values of the individual masses.

3 PLANETARY MIGRATION AND THE ADIABATIC INVARIANT THEORY

In principle, the results presented in the previous section should constitute a catalog of all corotational solutions in the 2/1 resonance. If all extrasolar planets in this resonance lie in apsidal corotations, then their orbits should be well represented in those figures. However, what evidence do we have that all exoplanets in the 2/1 commensurability are in fact in apsidal corotation? All dynamical analysis of the *GJ876* system predict such a configuration, so there is little doubt that these planets satisfy our assumption. However, it is not immediate that the same should be generally valid for all other systems.

If all resonant exoplanetary systems acquired there present orbits as a result of planetary migration, then an important test would be to check whether captured migrating bodies do exhibit apsidal corotations. Recent hydrodynamical simulations (e.g. Snellgrove et al. 2001, Kley 2003, Papaloizou 2003, etc.)) of the evolution of two planets immersed in a gaseous disk, have always shown corotational final orbits. Kley (2003), modeled *55 Cnc*, and capture occurred in the 3/1 commensurability. In the other two papers the simulated system was *GJ876* and trapping occurred in the 2/1 mean-motion resonance. Other works, such as Nelson and Papaloziou (2002) have included a modeled migration in the equations of motion of the planets, as constant perturbations in the angular momentum and orbital energy. Solving these equations numerically, they have also found corotations as a final result.

3.1 Numerical Simulations of Resonance Capture

In order to test whether these results are only valid for point values of disk parameters or even for certain types of driving mechanisms,

we performed a series of numerical simulations of the planetary migration. In this series, we studied the trapping process and posterior evolution of the system inside the resonance (general three-body problem) for a wide range of exterior non-conservative forces. Each force was modeled as an additional term added to Newton's equations of motion, and all runs simulated capture in the 2/1 mean-motion resonance. We adopted various types of forces, including: *(i)* Tidal interactions (Mignard 1981), *(ii)* interaction with a planetesimal disk (modeled according to Malhotra 1995) and, *(iii)* disk torques modeled similar to Nelson and Papalozoiu (2002). Most of these mechanisms can be reproduced as particular cases of a Stokes-type non-conservative force of the type:

$$\frac{d^2\mathbf{r}}{dt^2} = -C(\mathbf{v} - \alpha\mathbf{v}_c) \tag{8}$$

where $\mathbf{r}$ is the position vector of the body (reference frame centered in the star), $\mathbf{v}$ is its velocity vector and $\mathbf{v}_c$ is the circular velocity vector at the same point. Unlike usual Stokes drag where α is fixed by the characteristics of the gas, in this generic case both coefficients C and α can be taken as external parameters and varied in each run. The first is usually considered defined positive (i.e. $C > 0$) while the second can take any value. From Beaugé and Ferraz-Mello (1993) and Gomes (1995) it can be seen that, to first order and in the absence of additional gravitational perturbations, the effects of the force (8) in the semimajor axis and eccentricity are given by:

$$a(t) = a_0 \exp(-At) \qquad ; \qquad e(t) = e_0 \exp(-Et) \tag{9}$$

where a_0 and e_0 are the initial conditions at $t = 0$, and A,E are the inverse of the e-folding times in each orbital element. These quantities are given by:

$$A = 2C(1 - \alpha) \qquad ; \qquad E = C\alpha. \tag{10}$$

Thus, $\alpha = 1$ gives a non-conservative force that gives an exponential decrease in semimajor axis but no change in the eccentricity, analogous to Malhotra's (1995) model of planet-planetesimal interactions. Moreover, when $\alpha < 0$ the force acts to increase the value of the eccentricity, and the opposite occurs when $\alpha > 0$. This can be used to model different types of behavior noted in planet-disk interactions, depending on the preponderance of external Limblad or co-orbital resonances (see Goldreich and Sari 2003).

Finally, we can consider the e-folding times as input parameters of the simulation and deduce the coefficients accordingly:

$$C = \frac{1}{2}A + E \qquad ; \qquad \alpha = \frac{E}{C}. \tag{11}$$

With all these options we hope to have a fairly general idea of the capture process in the 2/1 resonance under a variety of conditions and physical models. Of course this list is not complete and it is not our intention to model all possible interactions, but it does give a general idea of the type of behaviors that can be expected.

3.2 Corotational Families as Evolutionary Tracks

Using these models for the driving mechanism, we performed a series of numerical simulations of the evolution (and resonance capture) of two planets with a given mass ratio, and initial circular orbits with $a_1 = 5.2$ AU and $a_1 = 8.5$ AU. These semi-major axes place the bodies outside, but close to, the 2/1 mean-motion resonance. In particular, we did several runs with a non-conservative force given by equation (8), and adopting different values of the e-folding times in the range $A \in [10^{-7}, 10^{-4}]$ and $E \in [10^{-11}, 10^{-4}]$.
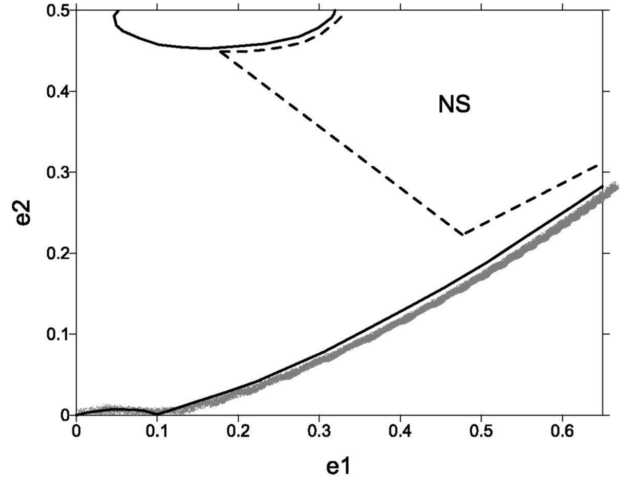

Figure 7. Relation between eccentricities of inner and outer planets during the orbital evolution inside the 2/1 resonance, using $m_2/m_1 = 1.5$. Grey symbols show the results of several numerical simulations with different values of $A \in [10^{-6}, 10^{-4}]$ and $E \in [10^{-11}, 10^{-4}]$. Black lines show the two families of zero-amplitude corotations for this mass ratio: asymmetric solutions for high values of e_2 and aligned orbital configurations in the other case.

In accordance with other works, all our runs ended in apsidal corotations. Although dynamical θ-librations have previously been detected in resonance trapping for the restricted three-body problem with $m_2 = 0$ (see Beaugé and Ferraz-Mello 1993), it seems not likely (or not possible) for the general planetary case in the 2/1 commensurability. Even though we cannot rule out their existence, if present they should have small trapping probabilities or be the consequence of very particular kind of dissipation forces. Perhaps future analytical work will solve this question categorically; however, and for the remainder of this work, we will assume that only apsidal corotations are available for trapped bodies.

Typical results (using $m_2/m_1 = 1.5$) are shown in Figure 7, where we have plotted the relationship between the eccentricities of the bodies prior to capture and during the orbital evolution inside the commensurability. The results of *all* numerical simulations are shown in gray symbols. Although different driving mechanisms may yield solutions which vary in capture timescales or amplitudes of libration, we can see that all points fall in the same region of the plane (e_1, e_2). In fact, since the initial orbits were circular, the gray symbols define an "evolutionary curve" of the system, in which the eccentricities evolve from the origin to the right-hand side of the graph as function of time. Some values of A, E yield equilibrium eccentricities (see Lee and Peale 2002), in which case the evolution stops at some critical value of (e_1, e_2). Conversely, for other values of the e-folding times, evolution continues until e_1 reaches quasi-parabolic values and both planets collide.

In the same figure we have also plotted the region of No Solutions and, in black continuous lines, the families of corotations for this mass ratio. Three families exist: asymmetric solutions lie on the top of the graph, for high values of e_2 and low values of e_1. Symmetric (aligned and anti-aligned) apsidal corotations lie in the lower half of the plot. Each individual solution of a given family is parametrized by different values of the total angular momentum. Note that the symmetric family shows a very good agreement with the numerical simulations of the evolution of the planets. This in fact shows that during the capture process, the system evolves adi-

abatically following the stable equilibrium solutions of the conservative system. Thus, the family of apsidal corotations does not only point the present possible locations of extrasolar planetary systems in the vicinity of the 2/1 resonance, but can also give information about the routes the bodies took from initially quasi-circular orbits towards their present locations.

This interpretation is possible as long as the driving mechanism of the migration is sufficiently slow, compared to the characteristic timescale of the conservative perturbations, so that the system can be well approximated by an smooth-varying Hamiltonian system. The Adiabatic Invariant Theory (e.g. Neishtadt 1975, Henrard 1982) shows that this is satisfied if the ratio between the period of oscillation around the apsidal corotation and the time derivative of the semimajor axis be much smaller than unity. In other words,

$$\varepsilon \equiv \tau A \ll 1. \tag{12}$$

In order to quantify this relation, let us recall equation (7), consider Jupiter-size planets and concentrate on the maximum values of τ. For initial semimajor axis a_2 in the vicinity of present day Jupiter, these results seem to indicate that adiabaticity is satisfied if the migration time-scale $1/A \gg 5 \times 10^3$ years for $m_2/m_1 = 0.5$ and $1/A \gg 8 \times 10^2$ years for $m_2/m_1 = 3$. For much smaller semimajor axes (e.g. $a_2 = 0.3$ AU), these numbers fall to values of the order of 10^2 years. Thus, any dissipative force with migration timescale much larger than this should be adiabatic and thus its evolution well modeled by the families of corotational solutions.

To date there is no concrete evidence as to the real duration of the migration, although it is believed to have taken between 10^5 and 10^7 years (see Trilling et al. 2002). If this is indeed the case, then the adiabatic approximation should be a good model, at least close to the central star. Even if these limits are too conservative and an extremely fast Type I migration dominated the evolution, the mechanism most probably had a smooth decay in magnitude with time, thus becoming much slower towards the end of the process. From some point on then, the mechanism should satisfy condition (12). If true, then it seems that any migration process should lead to present orbital configurations consistent with apsidal corotations.

In order to test this idea, Figure 8 shows, for each mass ratio discussed in the previous paragraphs, two simulations of the capture process, one with $A = 10^{-6}$ (black) and the other considering an extremely rapid migration: $A = 10^{-4}$ (gray). Results are shown in the eccentricity plane in both cases. For the two larger mass ratios both simulations follow practically the same routes, and are consistent with the corotational families. Recall that these mass ratios have small periods of oscillation and no asymmetric apsidal corotations. The top graphs show a different story. The system with $m_2/m_1 = 0.8$ shows fair agreement between both simulations for symmetric apsidal corotations, but completely different results for the asymmetric region. The results for $m_2/m_1 = 0.5$ are an extreme case. The fastest migration shows very little in common with the adiabatic evolution, although capture still takes place and both eccentricities continue to grow as function of time.

Figure 9 presents the evolution of both angular variables, as function of the growing e_1, for the two smallest mass ratios. Colors are the same as the previous figure, with black lines corresponding to the slowest (adiabatic) migration and gray to the fastest. We can see that a non-adiabatic force not only implies different evolutionary tracks in the eccentricity plane, but also in the angular variables. Interestingly, in both cases the largest dissipation causes very evident asymmetric apsidal corotations which have no association with the conservative equilibrium solutions. This seems to indicate that, perhaps, the lack of adiabaticity is also accompanied
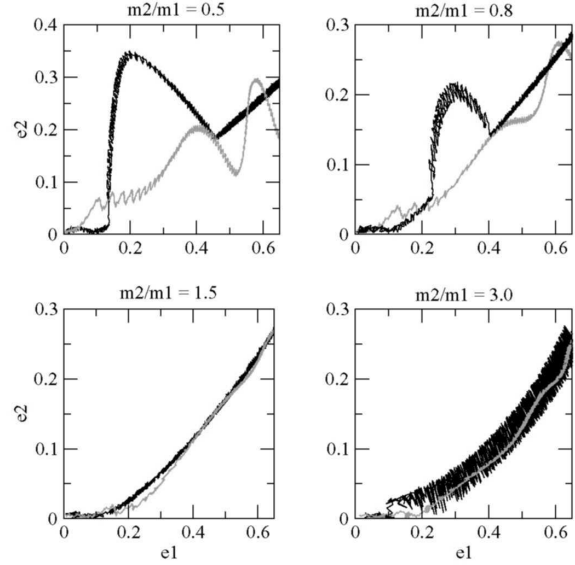

Figure 8. Numerical simulations of adiabatic migration (black symbols) and non-adiabatic (gray) for four different mass rations. All plots show the evolutionary tracks in the eccentricities plane.
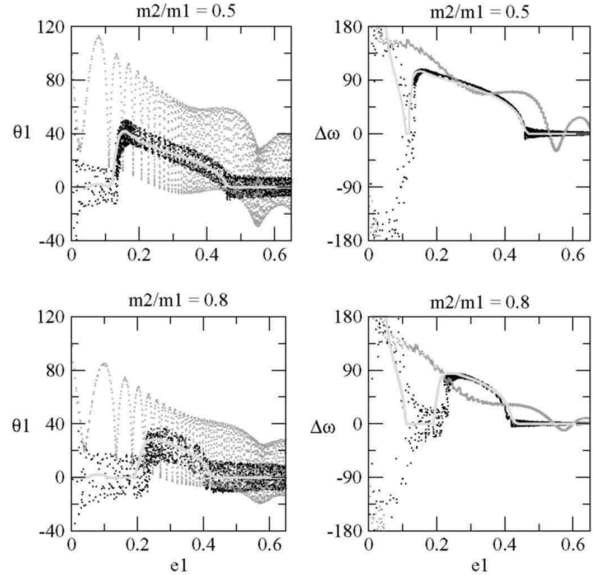

Figure 9. Same as before for only two mass-ratios, and showing the temporal evolution of the resonant angle θ_1 (left) and difference in longitudes of pericenter (right). The light-gray continuous lines show the analytical corotational solutions parametrized by the eccentricity of the inner planet.

by a change in the equilibrium solutions. Only for high values of e_1, accompanied by small values of the semimajor axes (due to the orbital decay) do both curves reasonably agree, consistent with a decrease in τ compared with the migration timescale.
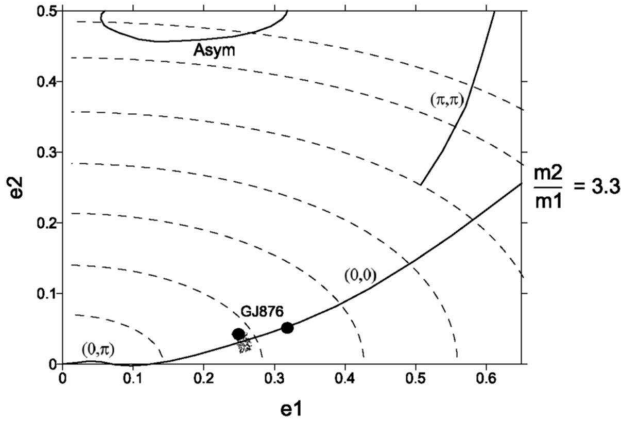
Figure 10. Corotational families for mass ratio equal to the *GJ876* system, together with current orbital fits (in full circles). Dotted curves mark the levels of constant total angular momentum.

4 CONFIRMED AND UNCONFIRMED PLANETARY SYSTEMS IN THE 2/1 RESONANCE

From the previous simulations we can conclude that for mass ratios larger than 1.5, even a fast planetary migration leads to evolutionary tracks consistent with our corotational families. We can then proceed to test this idea with the known planetary systems in the vicinity of the 2/1 resonance, be them confirmed or unconfirmed bodies. Recent stability analysis of several resonant exoplanets have shown that in some cases (e.g. *47 Uma*) a corotation is not the only dynamically stable configuration. In fact, apart from the well-studied case of *GJ876*, it is not absolutely certain whether any other real system is in an actual apsidal corotation. However, as deduced from the analysis of the previous section, planetary migration does seem to imply this type of solutions.

In view of this, a good test for the migration hypothesis is to check whether current orbital fits are *consistent* with apsidal corotations. If they are not, then we stand with two possibilities. Perhaps the orbital fits may not be adequate and need to be conferred. Conversely, if the data analysis is confirmed, then either these systems did not undergo migration at all, or this process was highly un-adiabatic. Either way, we can obtain important information of the formation process and posterior evolution of these planetary systems. Is it thus important to stress that our aim will not be to certify whether the current planets are in fact in apsidal corotation, but solely if the orbital fits are consistent with these configurations.

4.1 *Gliese 876*

We begin with the well known *GJ876* planets. Current information for this system gives two different possible orbits (see Laughlin and Chambers 2001, Lee and Peale 2002), depending on the chosen observational data base. Dynamical fits using Keck+Lick observations yield $(e_1, e_2) = (0.27, 0.10)$, while data from Keck alone gives $(e_1, e_2) = (0.33, 0.05)$. For both, however, the mass ratio is very similar, $m_2/m_1 \sim 3$. Figure 10 shows the different families of corotations for $m_2/m_1 = 3.3$ in the eccentricity plane. The real bodies are represented by filled circles. Note that both observational fits lie very close to the zero-amplitude solutions, and it is easy to deduce their evolutionary track from initially circular orbits. Thus, we can know that the planets were initially captured

in an anti-aligned corotation, but switched to an aligned orbit when e_1 surpassed the critical value $e_c = 0.1$.

Together with the present orbits, we have also plotted (in small dots) the temporal variation of the eccentricities during a 10^5 timespan. The Keck orbit is very close to the actual zero-amplitude corotation, so the dots are masked within the filled circle. The other orbit, however, shows a perceptible oscillation around the corotational family. This behavior can in fact be predicted from the invariance of the total angular momentum. Writing this explicitly from equation (4) and supposing that the magnitude in the temporal variation of the eccentricity is much larger than in semimajor axis, we find that the level curves of constant J_{tot} in the eccentricity plane are given by the expression:

$$L_1 \sqrt{1 - e_1^2} + L_2 \sqrt{1 - e_2^2} = const. \tag{13}$$

where the Delaunay momenta L_i can be fixed at exact resonance (see Zhou et al. 2004 for a similar analysis). The broken curves in Figure 10 show the level curves of this integral, for different values of the constant. Note that the variation of the eccentricities for the Keck+Lick data follows closely their trend.

4.2 *HD82943*

Although not many new results were obtained from this previous system, it is useful as an example where the adiabatic migration scenario yields results consistent with the observational orbital fits. The *HD82943* planets, however, are much more compromising.

Until very recently, the best observational data for this system was consistent with $m_2/m_1 = 1.9$ and eccentricities $(e_1, e_2) = (0.54, 0.41)$ (see Extrasolar Planets Encyclopedia homepage: www.obspm.fr/planets). Ji et al. (2003) showed that this configuration is only stable if both planets are trapped in a (π, π)-corotation. Figure 11a shows all the families of corotation for this mass ratio. Once again, the orbits fits are presented by a filled circle, and the levels of constant angular momentum by broken curves. Via a numerical integration, the temporal variation of the eccentricities are shown with small dots. We note a large-amplitude oscillation around the corotational family, although this system seems to be very stable over large timescales.

However, a problem arises when we try to deduce the evolutionary track of these planets from initially circular orbits. After performing numerous numerical integrations, we could not find a single initial condition or choice of parameters for the dissipative force that allowed a jump from the family of aligned corotations to the (π, π) case. Both families are not only disconnected, but are separated by a region not protected from collisions; thus there seems to be no road from one to the other that does not lead to a physical disruption of both planets.

In a recent communication, Lee and Peale (2003) argued that this orbital fit is not consistent with a smooth planetary migration, unless *(i)* the planets suffered a significant mass variation during the migration or, *(ii)* the orbits are not coplanar. The first alternative seems unlikely, since the collision curve does not depend on the masses. Thus, unless the planets were virtually insignificant at the time of the orbit intersection, they would still have suffered a physical encounter leading to a disruption or an ejection. The existence of a mutual inclination sufficiently large to avoid close encounters is also questionable, since there is no indication from cosmogonic theories that massive planets could form at highly non-planar orbits. Migration simulations by Thommes and Lissauer (2003) also show no inclination excitation for eccentricities below 0.65. The
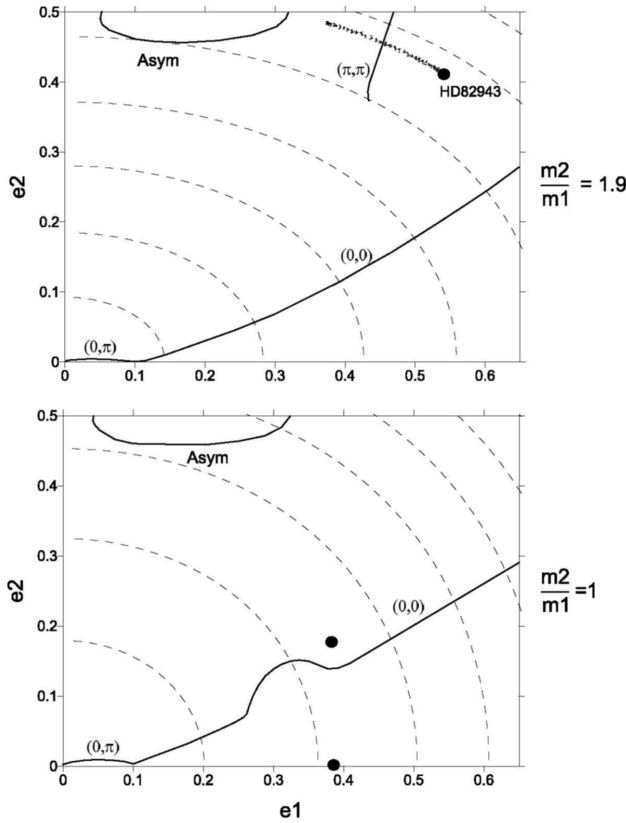
Figure 12. Numerical simulation of two stable initial conditions in the vicinity of the Mayor et al (2004) data for the *HD82943* planets. Left-hand plots show a corotation, while the right-hand a paradoxic θ_1-libration with a circulation of the difference of pericenter. See text for details.

Figure 11. Same as previous figure, but for the *HD82943* planets. Top graph corresponds to $m_2/m_1 = 1.9$ and eccentricities from data fits published in the Exoplanet encyclopedia homepage. Bottom: $m_2/m_1 = 1$ and orbital fits from Mayor et al (2004). Note: Currently, there is a general agreement that the orbital fit shown in the top graph is not correct. Thus, these results represent a "fictitious" system.

authors finally considered a different option, imagining that the capture into this type of corotation was obtained as a consequence of a close encounter of one of the bodies with a third planet. As a result, this other body was ejected and engulfed by the star. This alternative found certain support in new spectral analysis of the *HD82943* star showing evidence of Li^6 (Israelian et al. 2001), consistent with the recent absorption of a planet.

Two other alternatives also exist: either the planets are not in apsidal corotation at all (thus questioning the planetary migration scenario), or the orbital fit is not correct. A recent paper by Mayor et al. (2004) seems to indicate that the latter option may be true. The authors presented a new orbital fit, which yields $m_2/m_1 \simeq 1$ and eccentricities $(e_1, e_2) = (0.38, 0.18)$, thus significantly different from the previously published values. The same paper also shows a second new orbital fit, with the same mass ratio but $(e_1, e_2) = (0.38, 0.0)$. From their data analysis, both fits have similar residues, although there seems to be a marginal preference for the first.

Figure 11b shows our analysis of these new data. Note that for $m_2/m_1 = 1$ there is no (π, π) family, and the lower curve now has a hump corresponding to asymmetric solutions. Once again, both orbital sets are shown as filled circles. The first thing we note is that both fits are much more consistent with apsidal corotations than the results shown in the top graph. However, the solution $(e_1, e_2) = (0.38, 0.0)$ seems to be rather distant from its zero-amplitude curve. A simple analysis of the levels of con-
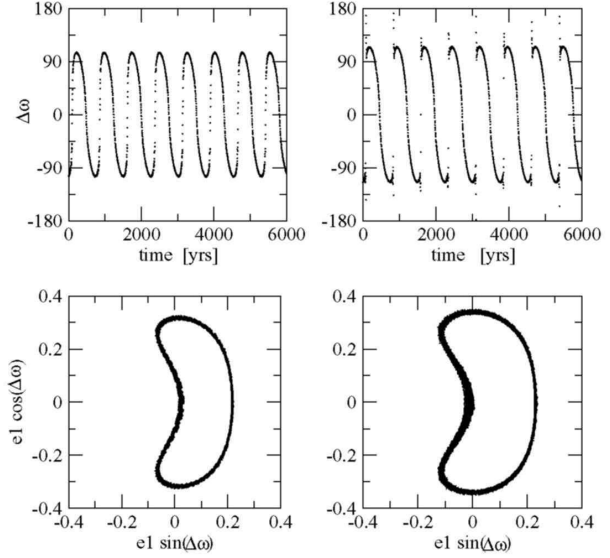
stant J_{tot} shows that, if this is an actual apsidal corotation, the temporal variation of the eccentricities would be very large. The $(e_1, e_2) = (0.38, 0.18)$ fit, however, is fairly close to the zero-amplitude curve, thus much more likely. However, in order to complete the analysis, we must also consider the values of the angular variables. From Figure 2 we obtain that a small-amplitude corotation for this point should have $\Delta\varpi \sim 50$ degrees. The actual value of this angle is 110 degrees, thus also indicating a large-amplitude apsidal corotation in the best case scenario.

In order to perform a more detailed study, we considered the Mayor et al. (2004) data for the planets including the uncertainties in each orbital element. We then generated a total of 100 initial conditions for the two planets according to a homogeneous distribution of orbital elements inside the error bars. Each resulting fictitious system was then integrated numerically for one million years. We found that 80% were unstable, ending in physical collisions or escape of one of the masses. Of the remaining, 15 were found in stable large-amplitude apsidal corotations (simultaneous libration θ_1 and $\Delta\varpi$), while 5 of the initial conditions yielded apparent θ_1-librations but with a circulation of the difference in longitudes of pericenter. Typical results are shown in Figure 12, where the left-hand plots correspond to a apsidal corotation and the right-hand shows one of the θ_1-librations. Top graphs present the temporal variation of $\Delta\varpi$ for a time interval of 6000 years, while the lower graphs show the orbits in the regular variables $(e_1 \cos \Delta\varpi, e_1 \sin \Delta\varpi$. We can clearly see that although the angle does circulate in the right-hand side, this is really topologically equivalent to the corotational solution. Moreover, the geometric average of the eccentricities in both solutions yield the same values $(e_1, e_2) = (0.35, 0.17)$, which is very close to the corotational family shown in Figure 11.

As a conclusion of this system, we see that sometimes a dynamical analysis of the present planetary orbits is not enough to ascertain that a given orbital determination is consistent with planetary migration. The evolutionary tracks can yield important information, and help identify problematic cases. Once noted, then we

can study whether the problem arises from orbital uncertainties, or if they point towards real dynamical evolution. Furthermore, the new orbital fit of this system is completely compatible with corotational solutions and, thus, with a planetary migration scenario.

4.3 *HD160691*

From the previous discussions, we can now divide the eccentricity plane into three distinct regions. These can be deduced from Figure 1. First, the NS (No Solution) region marks time-averaged values of the orbital eccentricities which are not allowed, unless the bodies are not in apsidal corotation, independent of any past migration. A second region groups the (π, π) and high-eccentricity asymmetric corotations. These solutions are not joined to quasi-circular orbits through continuous families. Thus, the presence of any real planetary system would be indication of a problematic case. It may indicate a catastrophic past (e.g. close encounters, ejection of missing planets, etc.) or be evidence that the planets are not in apsidal corotation at all. Once again, this would then be good evidence against planetary migration, at least in an adiabatic process. Finally, a third region (including aligned, anti-aligned and asymmetric apsidal corotations) shows those orbits whose evolutionary tracks are consistent with adiabatic migration.

As an example of the application of this plot, we analyze the *HD160691* system. According to (Jones et al. 2002), two planets orbit this star, with $m_2/m_1 = 0.6$, and $(e_1, e_2) = (0.31, 0.80)$. Dynamical analysis by Bois et al. (2003) confirms the long term stability of this fit in an apsidal corotation. Nevertheless, a look at Figure 12 and the mass level curves of Figure 3, shows that even if this orbital fit is consistent with a corotational solution, the evolutionary track for this mass ratio cannot explain these present eccentricities. Thus, once again, and similarly with the previous system, we have found a problematic case. However, (and once again), recent observations (Mayor et al., private communication) and new orbital fits gave raised severe doubts as to the actual existence of the *HD160691*c planet. Thus, it seems that this resonant system is probably just as artifact of insufficient observational data.

5 CONCLUSIONS

In this paper we have presented a new catalog of general corotational solutions for the 2/1 mean-motion resonance. Apart from the well-known aligned and anti-aligned solutions, we have also extended our knowledge of asymmetric configurations and have mapped the recently discovered (π, π)-corotations. Since these periodic orbits depend on the planetary masses only through the ratio m_2/m_1, and on the semimajor axes only by a_1/a_2, they are very general in nature and should be valid for any exoplanetary system showing two planets trapped in this commensurability.

The determination of the period of oscillation τ_c around these fixed points of the averaged problem shows that any migration mechanism with characteristic timescale satisfying

$$\frac{1}{a_i}\frac{da_i}{dt} \gg \frac{m_1 m_2}{a_i}\tau_c \qquad (14)$$

should be adiabatic. Thus, starting from quasi-circular orbits, the evolutionary track of the planets inside the resonance should be well reproduced by the families of apsidal corotations for that particular mass ratio. This, together with the fitted values of the orbital eccentricities, allow us to stipulate whether present orbits are consistent with apsidal corotations and, consequently, with a planetary

migration of the system from cosmogonic locations far from the star. In other words, we are able to suggest a simple test which relates the present orbits with restrictions on the properties of the formation process of resonant planets.

Application to the 2/1 resonance shows that, of the three published systems, *GJ876* satisfies all our conditions satisfactorily. The old orbits for the *HD82943* planets are not consistent with our hypothetical relation between corotation and migration. However, new orbital determinations for this system have yielded values very compatible with orbital decay, thus indicating that the problem is due to uncertainties in the fits themselves. Finally another problematic case, *HD160691*, also seems to be biased by errors in the orbital fits. In fact, the exterior planet of this system is probably not existent at all.

In view of these examples, it seems that there is not indication of the existence of any planetary system in the 2/1 commensurability which is in specific contradiction with the hypothesis of planetary migration. Recent results by Zhou et al. (2004) for the *55 Cnc* system trapped in the 3/1 mean-motion resonance show similar results. It will be very interesting to extend this analysis to all future resonant systems and see whether an adiabatic migration mechanism continues to pass all the tests.

REFERENCES

Beaugé, C. and Ferraz-Mello, S. (1993). Resonance trapping in the primordial solar nebula: the case of a Stokes drag dissipation. *Icarus*, **103**, 301-318.
Beaugé, C. and Michtchenko, T.A. (2003). Modeling the high-eccentricity planetary three-body problem. *MNRAS* , **341**, 760-770.
Beaugé, C., Ferraz-Mello, S. and Michtchenko, T.A. (2003). Extrasolar planets in mean-motion resonance: apses alignment and asymmetric stationary solutions. *ApJ.* , **593**, 1124-1133.
Bois, E., Kiseleva-Eggleton, L., Rambaux, N. and Pilat-Lohinger, E. (2003). Conditions of dynamical stability for the HD160691 planetary system. *ApJ.*, **598**, 1312-1320.
Colombo, G., Franklin, F.A. and Shapiro, I.I. (1974). On the formation of the orbit-orbit resonance of Titan and Hyperion. *AJ*, **79**, 61.
Fernandez, J.A. and Ip,W.-H. (1984). Some dynamical aspects of the accretion of Uranus and Neptune - The exchange of orbital angular momentum with planetesimals. *Icarus*, **58**, 109-120.
Hahn, J.M. and Malhotra, R. (1999). Orbital evolution of planets embedded in a planetesimal disk. *AJ*, **117**, 3041-3053.
Ferraz-Mello, S., Tsuchida, M. and Klafke, J.C. (1993). On symmetrical planetary corotations. *Cel. Mech. & Dynam. Astron.*, **55**, 25-55.
Ferraz-Mello, S., Beaugé, C. and Michtchenko, T.A. (2003). Evolution of migrating planet pairs in resonance. *Cel. Mech. & Dynam. Astron.*, **87**, 99-112.
Ferraz-Mello, S., Michtchenko, T.A. and Beaugé, C. (2004). Regular motions in extra-solar planetary systems. In Chaotic Worlds: From Order to Disorder in Gravitational N-Body Dynamical Systems (B.A.Steves eds.), NATO ASI series volume, Kluwer, in press.
Greenberg, R. (1987). Galilean satellites - evolutionary paths in deep resonance. *Icarus*, **70**, 334-347.
Goldreich, P. and Sari, R. (2003). Eccentricity evolution for planets in gaseous disks. *ApJ.*, **585**, 1024-1037.

Gomes, R. (1995). The effect of nonconservative forces on resonance lock: stability and instability. *Icarus*, **115**, 47-59.

Hadjidemetriou, J. (2001). Resonant periodic motion and the stability of extrasolar planetary systems. *Cel. Mech. & Dynam. Astron.*, **83**, 141-154.

Hadjidemetriou, J. and Psychoyos, D. (2003). Dynamics of extra-solar planetary systems: 2/1 resonant motion. In Galaxies and Chaos (G. Contopoulos and N. Voglis eds.). Lecture Notes in Physics. Springer-Verlag. 412-432.

Henrard, J. (1982). Capture into resonance - an extension of the use of adiabatic invariants. *Cel. Mech. & Dynam. Astron.*, **27**, 3-22.

Israelian, G., Santos, N., Mayor M. and Rebolo, R. (2001). Evidence for planet engulfment by the star HD82943. *Nature*, **411**, 163.

Ji, J., Liu, L., Kinoshita, H., Zhou, J., Nakai, H. and Li, G. (2003). The Librating Companions in HD 37124, HD 12661, HD 82943, 47 Ursa Majoris, and GJ 876: Alignment or Antialignment? *ApJ.*, **591**, L57-L60.

Jones, H.R.A., Butler, R.P., Marcy, G.W., Tinney, C.G, Penny, A.J., McCarthy, C. and Carter, B.D. (2002). Extra-solar planets around HD196050, HD216437 and HD160691. *MNRAS*, **337**, 1170.

Kley, W. (2001). On the migration of a system of protoplanets. *MNRAS*, **313**, L47-L51.

Kley, W. (2003). Dynamical evolution of planets in disks. *Cel. Mech. & Dynam. Astron.*, **87**, 85-97.

Laughlin, G. and Chambers, J.E. (2001). Short-term dynamical interactions among extrasolar planets. *ApJ*, **551**, L109-L113.

Laskar, J. (1991). Analytical framework in Poincaré variables for the motion of the Solar System. In: *Predictability, Stability and Chaos in N-Body Dynamical Systems* (A. Roy, Ed.). NATO Asi Series, Vol. B272, pp. 93-112, Plenum Press, New York.

Lee, M.H. and Peale, S.J. (2002). Dynamics and origin of the 2:1 orbital resonances of the GJ 876 planets. *ApJ*, **567**, 596-609.

Lee, M.H. and Peale, S.J. (2003). Diversity and origin of 2:1 orbital resonances in extrasolar planetary systems. *BAAS*, **35**, No.4, 12.06.

Mayor, M., Udry, S., Naef, D., Pepe, F., Queloz, D., Santos, N.C. and Burnet, M. (2004). The CORALIE survey for southern extra-solar planets. XII. Orbital solutions for 16 extra-solar planets discovered with CORALIE. *AA*, in press.

Malhotra, R. (1995). The origin of Pluto's orbit: implications for the Solar System beyond Neptune. *AJ*, **110**, 420-429.

Michtchenko, T.A. and Ferraz-Mello, S. (2001). Modeling the 5:2 mean-motion resonance in the Jupiter-Saturn planetary system. *Icarus*, **149**, 357-374.

Mignard, F. (1981). The evolution of the lunar orbit revisited. I. *Moon and Planets*, **20**, 301-315.

Moons, M. (1994). Extended Schubart averaging. *Cel. Mech. & Dynam. Astron.*, **60**, 173-186.

Morbidelli, A. (1991). Topological methods for the qualitative analysis of a numerical simulation close to a resonance. In Interrelations Entre la Physique et la Dynamique des Petis Corpos du Systeme Solaire, (D. Benest and Cl. Froeschle eds.). Editions Frontières, pp 133-157.

Neishtadt, A.I. (1975). Passage through a separatrix in a resonance problem with a slowly-varying parameter. *Prikl. Matem. Mekhun*, **39**, 621-632.

Nelson, R.P. and Papaloizou, J.C.B. (2002). Possible commensurabilities among pairs of extrasolar planets. *MNRAS*, **333**, L26-L30.

Papaloizou, J.C.B. (2003). Disc-planet interactions: migration and resonances in extrasolar planetary systems. *Cel. Mech. & Dynam. Astron.*, **87**, 53-83.

Snellgrove, M.D., Papaloizou, J.C.B. and Nelson, R.P. (2001). On disk driven inward migration of resonantly coupled planets with application to the system around GJ876. *AA*, **374**, 1092-1099.

Thommes, E.W. and Lissauer, J.L. (2003). Resonant inclination excitation of migrating giant planets. *ApJ*, **597**, 566-580.

Trilling, D.E., Lunine, J.I. and Benz, W. (2002). Orbital migration and the frequency of giant planet formation. *AA*, **394**, 241-251.

Yoder, C.F. (1979). How tidal heating in Io drives the Galilean orbital resonance locks. *Nature*, **279**, 767-770.

Zhou, L-Y., Lehto, H.J., Sun, Y-S. and Zheng, J-Q. (2004). The apsidal corotation in mean-motion resonance: the 55 Cancri as an example. *MNRAS*, submitted.